## Bridge CST - SN4

Extra Practice

## Quadratic Inequalities in One Variable

## Quadratic Inequalities

1. Convert to function form in order to determine the sign of the function. Refer to this at the end.
2. Change the inequality sign to an equal sign.
3. Solve the quadratic equation using the most appropriate method.
4. Sketch a graph showing the zeroes and the direction of the graph
5. Observe the inequality sign and follow the rules of when the function is positive or negative

TEACH. LEARN. HELP.

- Here are the solutions/solutions sets of quadratic inequalities
- The following slides will provide all steps for each question..

| \#1 a) | $[-2,3]$ \#1 b) | $]-6,2[$ |
| :---: | :---: | :---: |
| \#1 c) | $]-\infty, 3.5[\cup] 1, \infty[\# 1 \mathrm{~d})$ | $]-17,9[$ |
| \#1 e) | $]-\infty,-2] \cup[10, \infty] \#$ f) | $[-8,5]$ |
| \#1 g) | $]-9,1[$ |  |
| \#2 a) | $]-\infty,-200[\cup] 50, \infty[\# 2 \mathrm{~b})$ | $[-3,7]$ |
| \# 2 c) | $[-12,4]$ \#2 d) | $]-3.6,2.4[$ |
| \#2 e) | $]-\infty,-15] \cup[3, \infty[\# 2 \mathrm{f})$ | No solution |
| \#2 g) | \# \#2 h) | $]-\infty,-1] \cup[2, \infty[$ |
| \#2 i) | $[-9,1]$ |  |

What are the solutions of the inequality

1a)

$$
5(x-3)(x+2) \leq 0
$$

1a)

$$
\begin{aligned}
& 5(x-3)(x+2) \leq 0 \\
& 5(x-3)(x+2)=0 \\
& (x-3)(x+2)=0
\end{aligned}
$$

Change the inequality sign to an equal sign.

$$
\begin{array}{l|l}
\hline(x-3)=0 & (x+2)=0 \\
x=3 & x=-2
\end{array}
$$



Observe the inequality sign and follow the rules of when the function is positive or negative.
$5(x-3)(x+2) \leq 0$


Here, $\leq 0$ means

- Observe the graph
below the $x$ axis, and
- include the
x - intercepts (zeroes).

Solution set

$$
[-2,3]
$$

What are the solutions of the inequality

1b)

$$
-4(x-2)(x+6)>0
$$

1b)

$$
\begin{aligned}
& -4(x-2)(x+6)>0 \\
& -4(x-2)(x+6)=0 \\
& (x-2)(x+6)=0
\end{aligned}
$$

Change the inequality sign to an equal sign.

Solve the quadratic equation using the most appropriate method.

Here, factor method.

$$
\begin{array}{l|l}
(x-2)=0 & (x+6)=0 \\
x=2 & x=-6
\end{array}
$$


$-4(x-2)(x+6)>0$

Parameter "a" is negative (-4), therefore the graph is in the shape of an upside-down U .

Observe the inequality sign and follow the rules of when the function is positive or negative.


Here, > 0 means

- Observe the graph above the $x$ axis, and
- Do not include the $x$ - intercepts (zeroes).

Solution set
]-6,2[

What are the solutions of the inequality?

1c)

$$
x^{2}+2.5 x+4>7.5
$$

$$
\begin{aligned}
& x^{2}+2.5 x+4>7.5 \\
& x^{2}+2.5 x+4=7.5 \\
& x^{2}+2.5 x-3.5=0 \\
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& x=\frac{-2.5 \pm \sqrt{(2.5)^{2}-4(1)(-3.5)}}{2(1)} \\
& x=\frac{-2.5 \pm \sqrt{(6.25)+14}}{2} \\
& x=\frac{-2.5 \pm \sqrt{20.25}}{2}
\end{aligned}
$$

Solve the quadratic equation using
the most appropriate method.
Here, quadratic formula method.

Sketch a graph showing the zeroes ( -3.5 \& 1) and the direction of the graph.

$$
x^{2}+2.5 x+4>7.5
$$

Parameter " a " is positive ( +1 ), therefore the graph is in the shape of a $U$.
Solution set

$$
]-\infty, 3.5[u] 1, \infty[
$$

$$
\begin{aligned}
& x^{2}+2.5 x+4>7.5 \\
& x^{2}+2.5 x-3.5>0
\end{aligned}
$$

Here, > 0 means

- Observe the graph above the $x$ axis, and
- Do not include the $x$ - intercepts (zeroes).

What are the solutions of the inequality?

1d)

$$
5(x+4)^{2}<845
$$

1d)

$$
5(x+4)^{2}<845
$$

$$
5(x+4)^{2}=845
$$

$$
(x+4)^{2}=169
$$

> Solve the quadratic equation using the most appropriate method.
> Here, the perfect square method.

$$
\sqrt{(x+4)^{2}}=\sqrt{169}
$$

$$
\begin{array}{l|l}
(x+4)=-13 & (x+4)=13 \\
x=-17 & x=9
\end{array}
$$

Sketch a graph showing the zeroes (-17 \& 9) and the direction of the graph.


## $5(x+4)^{2}<845$

Parameter " a " is positive ( +5 ), therefore the graph is in the shape of a $U$.

Observe the inequality sign and follow the rules of when the function is positive or negative.


$$
\begin{aligned}
& 5(x+4)^{2}<845 \\
& 5(x+4)^{2}-845<0
\end{aligned}
$$

Here, $<0$ means

- Observe the graph below the x axis, and
- Do not include the $x$ - intercepts (zeroes).

Solution set

$$
]-17,9[
$$

What are the solutions of the inequality?

1e)

$$
5(x-4)^{2}+6 \geq 186
$$

| $5(x-4)^{2}+6 \geq 186$ |  |
| :---: | :---: |
| $5(x-4)^{2}+6=186$ |  |
| $5(x-4)^{2}=$ | Change the inequality sign to an equal lign. |
| $(x-4)^{2}=36$ |  |
| $\sqrt{(x-4)^{2}}=$ | the most appropriate method. Here, the perfect square method. |
| $(x-4)=-6$ | $(x-4)=6$ |
| $x=-2$ | $x=10$ |

Sketch a graph showing the zeroes (-2 \& 10) and the direction of the graph.


$$
5(x-4)^{2}+6 \geq 186
$$

Parameter " a " is positive ( +5 ), therefore the graph is in the shape of a $U$.

## What are the solutions of the inequality?

1f)

$$
-2 x^{2}-6 x+30 \geq-50
$$

1f)

$$
\begin{aligned}
& -2 x^{2}-6 x+30 \geq-50 \\
& -2 x^{2}-6 x+30=-50 \\
& x^{2}+3 x-15=25 \\
& x^{2}+3 x-40=0 \\
& (x+8)(x-5)=0
\end{aligned}
$$

Solve the quadratic equation using the most appropriate method.
Here, the factoring method.

$$
\begin{array}{l|l}
x+8=0 & x-5=0 \\
x=-8 & x=5
\end{array}
$$

1f)
Sketch a graph showing the zeroes ( -8 \& 5) and the direction of the graph.


## $-2 x^{2}-6 x+30 \geq-50$

Parameter "a" is negative (-2), therefore the graph is the shape of an upside-down $U$.

Observe the inequality sign and follow the rules of when the function is positive or negative.


$$
\begin{aligned}
& -2 x^{2}-6 x+30 \geq-50 \\
& -2 x^{2}-6 x+80 \geq 0
\end{aligned}
$$

Here, $>0$ means

- Observe the graph above the $x$ axis, and
- Include the xintercepts (zeroes).


## Solution set

$[-8,5]$

## What are the solutions of the inequality?

1g)

$$
0.5\left(x^{2}+8 x+7\right)<8
$$

1g)

$$
0.5\left(x^{2}+8 x+7\right)<8
$$

$$
0.5\left(x^{2}+8 x+7\right)=8
$$

$$
0.5 x^{2}+4 x+3.5=8
$$

$$
0.5 x^{2}+4 x-4.5=0
$$

Solve the quadratic equation using the most appropriate method. Here, the factoring method.

$$
x^{2}+8 x-9=0
$$

$$
(x+9) \mid(x-1)=0
$$

$$
(x+9)=0 \quad(x-1)=0
$$

$$
\begin{array}{l|l}
x=-9 & x=1
\end{array}
$$

## $0.5\left(x^{2}+8 x+7\right)<8$

Parameter " a " is positive ( +0.5 ), therefore the graph is in the shape of a U .


Observe the inequality sign and follow the rules of when the function is positive or negative.

$0.5\left(x^{2}+8 x+7\right)<8$
$0.5 x^{2}+4 x+3.5<8$
$0.5 x^{2}+4 x-4.5<0$

Here, < 0 means

- Observe the graph below the $x$ axis, and
- Include do not include the x intercepts (zeroes).

Solution set
]-9, $[$

## Find the solution set of the following inequality?

2a)

$$
-12(x-50)(x+200)<0
$$

2a)

$$
-12(x-50)(x+200)<0
$$

Change the inequality sign to an equal sign.

$$
\begin{aligned}
& -12(x-50)(x+200)=0 \\
& (x-50)(x+200)=0
\end{aligned}
$$

Solve the quadratic equation using the most appropriate method. Here, the factoring method.

$$
\begin{array}{l|l}
(x-50)=0 & (x+200)=0 \\
x=50 & x=-200
\end{array}
$$


$-12(x-50)(x+200)<0$

Parameter " a " is negative ( -12 ) , therefore the graph is in the shape of an upside-down $U$.


## Find the solution set of the following inequality?

2b)

$$
0.2(x-7)(x+3) \leq 0
$$

$$
\begin{gathered}
\begin{array}{c}
0.2(x-7)(x+3) \leq 0 \\
0.2(x-7)(x+3)=0 \\
(x-7)
\end{array} \\
(x+3)=0 \\
\hline(x-7)=0
\end{gathered} \begin{aligned}
& (x+3)=0 \\
& x=7
\end{aligned}
$$

Change the inequality sign to an equal sign.

Solve the quadratic equation using the most appropriate method. Here, the factoring method.


# $0.2(x-7)(x+3) \leq 0$ 

Parameter "a" is positive ( +0.2 ), therefore the graph is in the shape of a U .

## Observe the inequality sign and follow the

 rules of when the function is positive or negative.

$$
0.2(x-7)(x+3) \leq 0
$$

Here, $\leq 0$ means

- Observe the graph below the $x$ axis, and
- include the $x$ intercepts (zeroes).

Solution set

$$
[-3,7]
$$

Find the solution set of the following inequality?
2c)

$$
3(x+4)^{2}+10 \leq 202
$$

2c)

$$
\begin{aligned}
& 3(x+4)^{2}+10=202 \quad \text { Change the inequality } \text { sign to an equal slign. } \\
& 3(x+4)^{2}=192 \\
& (x+4)^{2}=64 \\
& \sqrt{(x+4)^{2}}=\sqrt{64} \\
& x+4=8 \\
& x=4 \\
& \text { Solve the quadratic equation using } \\
& \text { the most appropriate method. } \\
& \text { Here, the perfect square method. }
\end{aligned}
$$



## $3(x+4)^{2}+10 \leq 202$

Parameter "a" is positive (+3), therefore the graph is in the shape of a $U$.

Observe the inequality sign and follow the rules of when the function is positive or negative.


$$
\begin{aligned}
& 3(x+4)^{2}+10 \leq 202 \\
& 3(x+4)^{2}-192 \leq 0
\end{aligned}
$$

Here, $\leq 0$ means

- Observe the graph below the $x$ axis, and
- Include the $x$ intercepts (zeroes).


## Solution set

$$
[-12,4]
$$

## Find the solution set of the following inequality?

2d)

$$
100 x^{2}+120 x-200<664
$$

$$
100 x^{2}+120 x-200<664
$$

$$
100 x^{2}+120 x-200=664
$$

$$
100 x^{2}+120 x-864=0
$$

$$
25 x^{2}+30 x-216=0
$$

> Solve the quadratic equation using the most appropriate method. Here, the quadratic formula.

$$
a=25 \quad b=30 \quad c=-216
$$

$$
\begin{aligned}
a=25 \quad b=30 \quad c & =-216 \\
2 d & =\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
x & =\frac{-(30) \pm \sqrt{(30)^{2}-4(25)(-216)}}{2(25)} \\
x & =\frac{-30 \pm \sqrt{900+21600}}{50} \\
x & =\frac{-30 \pm \sqrt{22500}}{50} \\
x & =\frac{-30 \pm 150}{50}
\end{aligned}
$$

Solve the quadratic equation using the most appropriate method. Here, the quadratic formula.

$$
\begin{array}{l|l}
x=\frac{-30+150}{50} & x=\frac{-30-150}{50} \\
x=\frac{120}{50} & x=\frac{-180}{50} \\
x=2.4 & x=-36
\end{array}
$$



| 2d) |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $(-3.6,0)$ |  |  |  |  |
|  |  |  | $(2.4,0)$ |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Convert to function form in order to determine the sign of the function.

Observe the inequality sign and follow the rules of when the function is positive or negative.

$$
\begin{aligned}
& 100 x^{2}+120 x-200<664 \\
& 100 x^{2}+120 x-864<0
\end{aligned}
$$

Here, < 0 means

- Observe the graph below the $x$ axis, and
- Include the x intercepts (zeroes).

Solution set

$$
]-3.6,2.4[
$$

Find the solution set of the following inequality?
2e)

$$
2(x+15)(x-3) \geq 0
$$

De)

$$
\begin{array}{l|l}
2(x+15) & (x-3) \geq 0 \\
2(x+15) & (x-3)=0 \\
(x+15) & (x-3)=0 \\
\hline(x+15)=0 & (x-3)=0 \\
x=-15 & x=3
\end{array}
$$

Change the inequality sign to an equal sign.

Solve the quadratic equation using the most appropriate method. Here, the factoring method.
e)

Sketch a graph showing the zeroes ( -15 \& 3) and the direction of the graph.


## $2(x+15)(x-3) \geq 0$

Parameter " a " is positive ( +2 ), therefore the graph is in the shape of a $U$.

Observe the inequality sign and follow the rules of when the function is positive or negative.
$2(x+15)(x-3) \geq 0$
Here, $\geq 0$ means

- Observe the graph above the $x$ axis, and
- Include the x intercepts (zeroes).

Solution set

$$
]-\infty,-15] \cup[3, \infty[
$$

## Find the solution set of the following inequality?

2f)

$$
-50 x^{2}+60 x>20
$$

2f)

$$
\begin{aligned}
& -50 x^{2}+60 x>20 \\
& -50 x^{2}+60 x=20 \\
& -50 x^{2}+60 x-20=0 \\
& 5 x^{2}-6 x+2=0 \\
& a=5 \quad b=-6 \quad c=2
\end{aligned}
$$

Change the inequality sign to an equal sign.

Solve the quadratic equation using the most appropriate method. Here, the quadratic formula.

2f)

$$
a=5 \quad b=-6 \quad c=2
$$

$$
\begin{aligned}
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& x=\frac{-(-6) \pm \sqrt{(-6)^{2}-4(5)(2)}}{2(5)}
\end{aligned}
$$

$$
x=\frac{6 \pm \sqrt{36-40}}{10}
$$

The square root of a negative number is undefined, therefore are

$$
x=\frac{6 \pm \sqrt{-4}}{10}
$$

Sketch a graph showing the zeroes (no zeroes) and the direction of the graph.


## $-50 x^{2}+60 x>20$

- There are no zeroes
- Parameter " a " is negative $(-50)$, therefore the graph is in the shape of an upside-down $U$.

Convert to function form in order to determine the sign of the function.

Observe the inequality sign and follow the rules of when the function is positive or negative.

$$
\begin{aligned}
& -50 x^{2}+60 x>20 \\
& -50 x^{2}+60 x-20>0
\end{aligned}
$$

Here, > 0 means

- Observe the graph above the x axis
- There is no graph above the x - axis

Solution set
$\varnothing$

Find the solution set of the following inequality?
$2 \mathrm{~g})$

$$
-x^{2}-5 x<8
$$

$2 \mathrm{~g})$

$$
\begin{aligned}
& -x^{2}-5 x<8 \\
& \text { Change the inequality sign to an equal sign. } \\
& -x^{2}-5 x=8 \\
& -x^{2}-5 x-8=0 \\
& x^{2}+5 x+8=0
\end{aligned}
$$

$2 \mathrm{~g})$

$$
a=1 \quad b=5 \quad c=8
$$

$$
\begin{aligned}
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& x=\frac{-(5) \pm \sqrt{(5)^{2}-4(1)(8)}}{2(1)} \\
& x=\frac{-5 \pm \sqrt{25-32}}{2} \\
& x=\frac{-5 \pm \sqrt{-7}}{2}
\end{aligned}
$$

The square root of a negative number is undefined, therefore there are no zeroes

Sketch a graph showing the zeroes (no zeroes) and the direction of the graph.


## $-x^{2}-5 x<8$

- There are no zeroes
- Parameter "a" is negative (-1), therefore the graph is in the shape of an upside-down $U$.


## $2 \mathrm{~g})$



Convert to function form in order to determine the sign of the function.

Observe the inequality sign and follow the rules of when the function is positive or negative.

$$
\begin{aligned}
& -x^{2}-5 x<8 \\
& -x^{2}-5 x-8<0
\end{aligned}
$$

Here, < 0 means

- Observe the graph below the x axis
- The complete graph is below the $x$ - axis

Solution set


## Find the solution set of the following inequality?

2h)

$$
2 x^{2} \geq 2 x+4
$$

$$
\begin{aligned}
& 2 x^{2} \geq 2 x+4 \\
& 2 x^{2}=2 x+4 \\
& 2 x^{2}-2 x-4=0 \\
& x^{2}-x-2=0 \\
& (x-2)(x+1)=0 \\
& \begin{array}{l|l}
x-2=0 & x+1=0
\end{array} \\
& x=2 \quad x=-1
\end{aligned}
$$

Solve the quadratic equation using the most appropriate method. Here, the factoring method.

Sketch a graph showing the zeroes and the direction of the graph.


Parameter "a" is positive (+2), therefore the graph is in the shape of a $U$.


$$
\begin{aligned}
& 2 x^{2} \geq 2 x+4 \\
& 2 x^{2}-2 x-4 \geq 0
\end{aligned}
$$

Here, $\geq 0$ means

- Observe the graph above the $x$ axis, and
- Include the x intercepts (zeroes).

Solution set
$]-\infty,-1] \cup[2, \infty[$

## Find the solution set of the following inequality?

2i)

$$
0.5(x+4)^{2}-12.5 \leq 0
$$

2i)

$$
\begin{aligned}
& 0.5(x+4)^{2}-12.5 \leq 0 \\
& 0.5(x+4)^{2}-12.5=0 \\
& 0.5(x+4)^{2}=12.5 \\
& (x+4)^{2}=25 \\
& \sqrt{(x+4)^{2}}=\sqrt{25} \\
& \begin{array}{l|l}
x+4=5 & x+4=-5 \\
x=1 & x=-9
\end{array}
\end{aligned}
$$

Sketch a graph showing the zeroes (-9 \& 1) and the direction of the graph.


## $0.5(x+4)^{2}-12.5 \leq 0$

Parameter " a " is positive ( +12.5 ), therefore the graph is in the shape of a U .


$$
0.5(x+4)^{2}-12.5 \leq 0
$$

Here, $\leq 0$ means

- Observe the graph below the $x$ axis, and
- Include the $x$ - intercepts (zeroes).


## Solution set

$$
[-9,1]
$$

